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The Voigt–Reuss–Hill Approximation and Elastic Moduli of Polycrystalline MgO, CaF_2, β-ZnS, ZnSe, and CdTe

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The Voigt–Reuss–Hill (VRH) approximation, a useful scheme by which anisotropic single-crystal elastic constants can be converted into isotropic polycrystalline elastic moduli, is shown to apply for moderately anisotropic cubic crystals like MgO, CaF_2, β-ZnS, ZnSe, and CdTe. Experimental values of polycrystalline isotropic elastic moduli for these materials are presented here, and the validity of the VRH approximation is established. The VRH approximation is then discussed for these materials with respect to their elastic anisotropy of crystals. To provide further support to this work, a numerical confirmation on the VRH moduli is made with the use of a high-speed computer by calculating the mean velocity of sound in crystals and comparing this result with the corresponding quantity calculated from the actual polycrystalline elastic moduli. The general agreement is observed.

1. INTRODUCTION

The Voigt–Reuss–Hill approximation is an averaging scheme by which anisotropic single-crystal elastic constants can be converted into isotropic polycrystalline elastic moduli. The averaging scheme is explicitly based on three independent theoretical contributions due to Voigt,[1] Reuss,[2] and Hill[3]; thus it was named the Voigt–Reuss–Hill (VRH) approximation.[4,5] The VRH approximation is simple and easily tractable in theory. For a single-phase crystalline aggregate made of crystals that are slightly anisotropic, the approximation gives the realistic values of isotropic elastic moduli. However, for aggregates containing crystals of highly anisotropic crystals, a question arises whether or not this approximation gives still the realistic estimates of the polycrystalline elastic properties.

The purpose of this paper is to study the VRH approximation for highly anisotropic cubic crystals and to examine to what extent this approximation is useful for calculating the polycrystalline isotropic moduli from the corresponding single-crystal data. The materials chosen for the present work are MgO, CaF_2, β-ZnS, ZnSe, and CdTe for which the well-characterized polycrystalline specimens were available. The polycrystalline β-ZnS, ZnSe, and CdTe are of particular interest in the present work since they are aggregate examples of highly anisotropic crystals having the same structure, and thus the measurements of the isotropic elastic moduli on these materials provide important empirical data that test the validity of the VRH approximation for high elastic anisotropy.

2. EXPERIMENTAL PROCEDURE

2.1. Specimens

The specimens used in the present work are vacuum hot-pressed polycrystalline aggregates and they are virtually free from porosity. These materials are known under a trade name Irtran, available from the Eastman Kodak Company. The specimens were about 0.635 by 0.953 by 7.620 cm in dimension, sufficiently large enough to make accurate resonance measurements as well as measurements of sound velocities. The faces of each specimen were polished to give a rectangular prism whose sides and ends are square and parallel to ±0.001 cm. Using ultrasonic velocity measurements, each specimen was checked for the elastic isotropy by rotating the transducer crystals on the specimen surface. All the specimens were found to be isotropic for both the longitudinal and shear waves. In the following, a brief description of these specimens is made under each material headings.

A. Polycrystalline MgO

One specimen designated hereafter as IR–MgO is a typical Irtran No. 5 material. The measured density was 3.5819 (±0.0009) g/cm^3 at 298°K and it should be compared with the x-ray density of 3.581 in the same unit. The chemical purity of IR–MgO was 99.98% MgO. Minor constituents contained in the specimen were as follows: Si 80, Ca 30, Al 25, Fe 25, Cu 10, Sn 5, Ba less than 3, B less than 10, Ni 5, Na 3, Li 2, Cr 4, K and Sr each less than 1 ppm. The specimen was highly translucent in the visible region, and it had an optical reflectance better than 60% in the 14 to 26 μ range. The measured reststrahlen lattice wavelength was 25.3 (±0.2) μ.[6]

B. Polycrystalline CaF_2

One specimen designated under IR–CaF_2 is a typical of Irtran No. 3 material. The measured density for the

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[1] W. Voigt, *Lehrbuch der Kristallphysik* (B. B. Teubner, Leipzig, 1928), p. 739.

[2] A. Reuss, Z. Angew. Math. Mech. **9**, 49 (1929).

[3] R. Hill, Proc. Phys. Soc. (London) **65**, 349 (1952).

[4] D. H. Chung, Phil. Mag. **8**, [89] 833 (1963).

[5] O. L. Anderson, J. Phys. Chem. Solids **24**, 909 (1963). The VRHG (Voigt–Reuss–Hill–Gilvarry) approximation so called by Anderson is exactly the same as the VRH (Voigt–Reuss–Hill) approximation. See, for example, *Physical Acoustics*, edited by W. P. Mason, (Academic Press, New York, 1965), Vol. III-B, Chap. 2.

[6] *Kodak Irtran 5 Material* (Eastman Kodak Company, Rochester, New York, 1963).

TABLE I. Measured isotropic elastic moduli of polycrystalline MgO, CaF_2, β-ZnS, ZnSe, and CdTe.

Specimen	Density (g/cm^3)	Elastic modulus[a] Young's	Shear	Longitudinal	Method of measurements
IR–MgO	3.5819(±0.0009)	30.72(±0.12)	12.93(±0.10)	···	Resonance
		···	12.90(±0.03)	33.83(±0.03)	Pulse superposition
IR–CaF_2	3.1792(±0.0008)	10.75(±0.09)	4.07(±0.07)	···	Resonance
		···	4.11(±0.03)	14.87(±0.03)	Pulse superposition
IR–ZnS	4.0791(±0.0009)	8.38(±0.09)	3.18(±0.07)	···	Resonance
		···	3.17(±0.03)	11.98(±0.03)	Pulse superposition
IR–ZnSe	5.2664(±0.0009)	7.46(±0.10)	2.81(±0.07)	···	Resonance
			2.89(±0.03)	9.83(±0.03)	Pulse superposition
IR–CdTe	5.8520(±0.0008)	···	1.40(±0.05)	6.07(±0.06)	Phase comparison
		···	1.38(±0.03)	6.08(±0.03)	Pulse superposition

[a] All values are at 298°K. All moduli are in units of 10^{11} dyn/cm^2.

IR–CaF_2 was 3.1792 (±0.0008) g/cm^3 at 298°K, and this value compares well with an x-ray density of 3.179 g/cm^3 at 298°K. The chemical purity of the specimen was 99.92% CaF_2, and a spectrochemical analysis showed the following impurities: Sr 700, Na 50, Mg 20, Si 3, and Mn 2 ppm with trances of Al, Ba, K, and Li. The IR–CaF_2 was highly translucent in the visible region as in the case of IR–MgO and it had the optical properties corresponding to a single-crystal CaF_2.

C. *Polycrystalline β-ZnS*

The β-ZnS specimen used in the present program is a typical of Irtran No. 2 material. The measured density was 4.0791 (±0.0009) g/cm^3 at 298°K, and this should be compared with a x-ray density of 4.088 g/cm^3 at the same temperature. The specimen has been referred to as IR–ZnS in the text, and it has a chemical purity of 99.997% ZnS. Among the impurities detected were Si 1 and Pb 1 ppm with traces of Cu, Fe, and Mg. X-ray diffraction patterns indicate the IR–ZnS was composed primarily of β-ZnS (i.e., sphalerite).

D. *Polycrystalline ZnSe*

One specimen of polycrystalline ZnSe used in the present program is a typical Irtran No. 4 material, and this has been designated as IR–ZnSe. The IR–ZnSe had the bulk density of 5.2664 (±0.0009) g/cm^3 at 298°K and this value compares well with 5.267 gm/cm^3, the x-ray density at the same temperature. This specimen has a chemical purity of 99.995% ZnSe, with minor constituents including Mg 5, Cu 3, B 10, and Al 2 ppm and traces of Cr and Ni. The IR–ZnSe was translucent in the visible region and had the optical properties similar to single-crystal ZnSe.

E. *Polycrystalline CdTe*

One specimen referred hereafter to as IR–CdTe is a typical of Irtran No. 6 material. The measured density was 5.8520 (±0.0008) g/cm^3 at 298°K and this may be compared with 5.854 gm/cm^3 calculated from the lattice constant[7] of 6.5815 Å at 298°K. The chemical purity of the specimen was 99.98% CdTe and the specimen contained the following impurities: B 10, Si 10, Mg 3 ppm and traces of Al, Ag, and Cu.

2.2. Measurements of Isotropic Elastic Moduli

The present work utilizes primarily a modified Förster-type resonance method in the kilocycle range[8] to determine the isotropic shear modulus and then Young's modulus of a bar-shaped polycrystalline specimen. Two ultrasonic methods[9] (phase-comparison and pulse-superposition techniques due to McSkimin) often used in single-crystal measurements are also used, as complementary methods, for determining the elastic parameters of polycrystalline solids under investigation. Since a detailed description on all of these methods are found in the literature, this description is not reproduced here.

3. EXPERIMENTAL RESULTS

Table I lists the measured elastic moduli for all the polycrystalline specimens considered in the present work. For a given material, two sets of isotropic elastic moduli are entered. One set is the result obtained from the resonance method in the audio-frequency range and

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[7] P. W. Davis and T. S. Shilliday, Phys. Rev. **118**, 1020 (1960).

[8] S. Spinner and W. E. Tefft, Proc. ASTM **61**, 1221 (1961).

[9] H. J. McSkimin, *Physical Acoustics*, W. P. Mason, Ed. (Academic Press Inc., New York, 1964), Vol. I-A, Chap. 4.

TABLE II. Literature values of single-crystal elastic constants of MgO, CaF_2, β-ZnS, ZnSe, and CdTe.[a]

Material and reference temperature (°K)	Density (g/cm³)	s_{11}	$-s_{12}$	s_{44}	c_{11}	c_{12}	c_{44}	A	Reference
			($\times 10^{13}$ cm²/dyn)			($\times 10^{-11}$ dyn/cm²)			
MgO (300)	3.576	4.024	0.936	6.461	28.93	8.77	15.477	1.54	36D1[b]
MgO (298)	3.580	(4.075)	(0.950)	(6.757)	28.60	8.70	14.80	1.49	55B1[s]
MgO (293)	3.598	3.983	1.000	6.349	30.20	10.10	15.75	1.57	61S1[c]
MgO (300)	3.579	4.029	0.942	6.468	28.94	8.82	15.46	1.54	63C1[d]
MgO (298)	3.579	(3.988)	(0.969)	(6.405)	29.708	9.536	15.613	1.55	65B1[e]
MgO (297)	3.581	4.004	0.973	6.428	29.60	9.51	15.557	1.55	65C1[f]
CaF_2 (300)	3.180	(6.904)	(1.479)	(29.586)	16.4	4.47	3.38	0.57	28V1[g]
CaF_2 (298)	3.180	(7.097)	(1.660)	(28.814)	16.44	5.02	3.47	0.61	55B1[s]
CaF_2 (0)	3.180	(6.818)	(1.515)	(25.000)	16.8	4.8	4.0	0.67	58S1[h]
CaF_2 (300)	3.1795	(7.241)	(1.768)	(29.674)	16.4	5.3	3.37	0.61	60H1[i]
CaF_2 (293)	3.1804	(6.901)	(1.463)	(29.481)	16.357	4.401	3.392	0.57	63H1[j]
β-ZnS (298)	4.102	(19.429)	(7.309)	(22.936)	9.42	5.68	4.36	2.33	18V1[k]
β-ZnS (298)	4.102	(19.995)	(8.016)	(24.272)	10.79	7.22	4.12	2.31	44B1[l]
β-ZnS (298)	4.102	(20.497)	(8.075)	(29.412)	10.0	6.5	3.4	1.94	51P1[m]
β-ZnS (302)	4.079	(18.817)	(7.090)	(22.173)	9.76	5.90	4.51	2.34	63E1[n]
β-ZnS (298)	4.088	(18.381)	(7.065)	(21.678)	10.46	6.53	4.613	2.35	63B1[o]
β-ZnS (293)	4.083	(18.706)	(7.201)	(21.645)	10.32	6.46	4.62	2.39	63Z1[q]
ZnSe (298)	5.262	(22.570)	(8.486)	(22.676)	8.10	4.88	4.41	2.74	63B1[o]
CdTe (298)	5.854	(42.542)	(17.338)	(50.151)	5.351	3.681	1.994	2.39	62M1[r]
CdTe (77)	5.860	(38.296)	(15.758)	(51.020)	6.15	4.30	1.96	2.12	63B1[o]

[a] Values in parentheses are the present writers' computation based on work of the original authors. The number of the significant figures does not necessarily represent the accuracy of the values.

[b] 36D1: M. A. Durand, Phys. Rev. **59**, 449 (1936).

[c] 61S1: C. Susse, J. Res. CNSR (Paris), **54**, 23 (1961).

[d] 63C1: D. H. Chung, J. J. Swica, and W. B. Crandall, J. Am. Ceram. Soc. **46**, 452 (1963).

[e] 65B1: E. Hal Bogardus, J. Appl. Phys. **36**, 2504 (1965).

[f] 65C1: D. H. Chung (unpublished work, 1965).

[g] 28V1: W. Voigt *Lehrbuch der Kristallphysik*, (Teubner, Berlin, 1928), p. 744.

[h] 58S1: R. Srinivasan, Proc. Phys. Soc. (London), **72**, 556 (1958).

[i] 60H1: D. R. Huffman and M. H. Norwood, Phys. Rev. **117**, 709 (1960).

[j] 63H1: S. Haussuhl, Phys. Status Solidi **3**, 1072 (1963).

[k] 18V1: W. Voigt, Göttinger Nachr. **424** (1918).

[l] 44B1: S. Bhagavantam and D. Suryanarayana, Proc. Ind. Acad. Sci., **A20**, 304 (1944).

[m] 51P1: E. Prince and W. A. Wooster, Acta Cryst. **4**, 191 (1951).

[n] 63E1: N. G. Einspruch and R. J. Manning, J. Acoust. Soc. Am. **35**, 215 (1963).

[o] 63B1: D. Berlincourt, H. Jaffe, and L. R. Shiozawa, Phys. Rev. **129**, 1009 (1963).

[q] 63Z1: A. Zarembovitch, J. Phys. (Paris) **24**, 1097 (1963).

[r] 62M1: H. J. McSkimin and D. G. Thomas, J. Appl. Phys. **33**, 56 (1962).

[s] 55B1: S. Bhagavantam, Pro. Ind. Ind. Acad. Sci., **A41**,: 78 (1965).

the other from McSkimin's pulse-superposition method in the ultrasonic frequency range. Because of high damping, the resonance measurements on IR–CdTe were not made but instead sound velocities were measured by both the pulse-superposition and phase-comparison methods. The primary constants resulting from the resonance method are the shear and Young's moduli, whereas those from the ultrasonic methods are the shear and longitudinal moduli. In each case, the combined experimental errors in the modulus are indicated. It is noted here that the elastic moduli determined from the resonance method are always in the general agreement with the ones determined from the ultrasonic methods.

4. SINGLE-CRYSTAL ELASTIC CONSTANTS AND CALCULATIONS OF THE VOIGT–REUSS–HILL (VRH) MODULI

For most materials considered here, there are more than one set of single-crystal elastic constants reported in the literature and in some cases they differ in values from one author's result to another. This situation presents a problem of selecting the elastic constants realistic of the solid. For this reason, all the known values of elastic constants are quoted in Table II as they found in the literature. In each case, entries have been made for the reference temperature after the material heading and also for the density of the specimen used by the original author, the elastic constants, the elastic anisotropy factor, and the source of this information. Using these single-crystal data, the isotropic VRH moduli are calculated according to the relations

$$K_{\mathrm{VRH}}=K_V=K_R=K^*=c_{11}-2C/3 \tag{1}$$

and

$$G_{\mathrm{VRH}}=G^*=(G_V+G_R)/2, \tag{2}$$

where $C=(c_{11}-c_{12})$. The limiting moduli G_V and G_R are given by

$$G_V=C/5+3c_{44}/5, \tag{3}$$

and

$$G_R=5Cc_{44}/(4c_{44}+3C), \tag{4}$$

TABLE III. The Voigt–Reuss–Hill moduli of MgO, CaF_2, β-ZnS, ZnSe, and CdTe.[a]

Material and reference temperature (°K)	Reference to c_{ij}[b]	K_{VRH}	G_{VRH}	E_{VRH}	$G_V - G_R$	$E_V - E_R$
MgO (300)	36D1	15.490	13.033	30.532	0.571	1.045
MgO (298)	55B1	15.333	12.623	29.712	0.475	0.877
MgO (293)	61S1	16.800	13.154	31.291	0.632	1.194
MgO (300)	63C1	15.530	13.013	30.513	0.573	1.051
MgO (298)	65B1	16.260	13.104	30.985	0.596	1.112
MgO (297)	65C1	16.205	13.056	30.873	0.595	1.109
CaF_2 (300)	28V1	8.447	4.251	10.920	0.325	0.716
CaF_2 (298)	55B1	8.827	4.241	10.965	0.250	0.557
CaF_2 (0)	58S1	8.800	4.708	11.985	0.185	0.399
CaF_2 (300)	60H1	9.000	4.120	10.723	0.244	0.550
CaF_2 (293)	63H1	8.386	4.264	10.936	0.324	0.712
β-ZnS (298)	18V1	6.930	3.112	8.114	0.520	1.180
β-ZnS (298)	44B1	8.410	2.945	7.908	0.481	1.157
β-ZnS (298)	51P1	7.667	2.604	7.017	0.271	0.656
β-ZnS (302)	63E1	7.187	3.208	8.372	0.539	1.226
β-ZnS (298)	63B1	7.840	3.276	8.619	0.556	1.286
β-ZnS (293)	63Z1	7.747	3.255	8.558	0.578	1.333
ZnSe (298)	63B1	5.953	2.945	7.573	0.689	1.524
CdTe (298)	62M1	4.238	1.406	3.796	0.248	0.604
CdTe (77)	63B1	4.917	1.450	3.959	0.192	0.477

[a] All moduli are in units of 10^{11} dyn/cm².

[b] See Table II for the complete references.

respectively. Knowing the bulk modulus K^* and the shear modulus G^*, one can calculate Young's modulus E^*, the longitudinal modulus L^*, and Lame's constant λ^* as well as the compressibility χ^* and Poisson's ratio μ^* from the well-known relations[10] of the isotropic elasticity.

The result of these calculations for the isotropic bulk, shear, and Young's moduli are tabulated in Table III for all the single-crystal data listed in Table II. Also entered are the differences in the limiting Voigt and Reuss moduli since they are related to the actual magnitude of the elastic anisotropy possessed by crystals.[11] Table IV compares selected values of the measured and calculated isotropic moduli from Table I and Table III, respectively.

5. DISCUSSION

It is apparent from Table IV that the measured polycrystalline elastic moduli are in good agreement with the isotropic VRH moduli calculated from the corresponding single-crystal elastic constants. The differences observed between the measured and calculated values are in most cases within the scatters in the calculated VRH moduli resulting from the differences in the single-crystal elastic constants of one author to another (see Table III). This indicates the success of

TABLE IV. Comparison between the measured and calculated isotropic elastic moduli of MgO, CaF_2, β-ZnS, ZnSe, and CdTe.

Material and references[b]		Elastic modulus[a] Shear	Young's	Bulk
MgO	Single-crystal (65C1)	13.056	30.873	16.205
	polycrystalline	12.93(±0.10)	30.72(±0.12)	16.41
CaF_2	Single-crystal (60H1)	4.120	10.723	9.000
	polycrystalline	4.07(±0.07)	10.75(±0.09)	9.98
β-ZnS	Single-crystal (63E1)	3.208	8.372	7.187
	polycrystalline	3.18(±0.07)	8.38(±0.09)	7.66
ZnSe	Single-crystal (63B1)	2.945	7.573	5.953
	polycrystalline	2.88(±0.07)	7.46(±0.10)	6.67
CdTe	Single-crystal (62M1)	1.406	3.796	4.238
	polycrystalline	1.38(±0.03)	3.73	4.25

[a] All moduli are in units of 10^{11} dyn/cm².

[b] See Table II for the complete references.

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[10] F. Birch, J. Geophys. Res. 65, 3855 (1960); Table 4.

[11] D. H. Chung and W. R. Buessem, J. Appl. Phys. (in press).

the VRH approximation, even for the anisotropic crystals like CdTe and ZnSe.

To examine a possible dependence of the VRH approximation on elastic anisotropy, we plotted the ratio of the measured modulus to the calculated VRH modulus as a function of percent elastic anisotropy in Figs. 1 and 2. The percent elastic anisotropy[11] referred here is

$$A^*(\text{in } \%) = [3(A-1)^2/3(A-1)^2+25A] \times 100, \quad (5)$$

where $A = 2c_{44}/(c_{11}-c_{12})$. Note that the limiting Voigt and Reuss moduli result in the wider spread as the elastic anisotropy of crystal becomes large. It is noteworthy, however, that the measured moduli lie within the spread for every crystals considered in the present program. As seen in Fig. 2, the ratio of shear moduli $(G_{\text{meas}}/G_{\text{VRH}})$ is smaller than unity in all cases and the deviation of this ratio from unity becomes large as the elastic anisotropy of crystal increases. Similar observation can be made also for the case of Young's modulus. The trend of this deviation with increasing elastic anisotropy suggests that, for highly anisotropic crystals like Li and RbI, the VRH approximation may not be the good procedure to follow. But, for the cubic crystals possessing low or moderate elastic anisotropies (i.e., $A^* < 10\%$), the VRH approximation is believed to be accurate in giving the probable isotropic elastic moduli and these VRH moduli are as good as ones we measure in the laboratory.

TABLE V. Mean velocity of sound for MgO, CaF_2, β-ZnS, CdTe, and ZnSe.[a]

Materials and reference[b]		A^* (%)	v_m [Eq. (6)]	v_m^* [Eq. (7)]
MgO	Single-crystal (65C1)	2.28	6.617	6.654
	polycrystalline		...	6.626
CaF_2	Single-crystal (60H1)	2.96	4.001	4.022
	polycrystalline		...	4.006
β-ZnS	Single-crystal (63E1)	8.49	3.122	3.135
	polycrystalline		...	3.127
CdTe	Single-crystal (62M1)	8.83	1.712	1.743
	polycrystalline		...	1.727
ZnSe	Single-crystal (63B1)	11.70	2.406	2.637
	polycrystalline		...	2.614

[a] All values of the velocity are in units of 10^5 cm/sec.

[b] See Table II for the complete references.

To provide an additional support to this conclusion, we take a numerical approach in which we calculate the mean velocity of sound in a given crystal and then compare this result with the corresponding quantities deduced from the Debye continuum relation. The mean velocity of sound in an anisotropic crystal is

$$v_m = \left[\frac{1}{3} \sum_{j=1}^{3} \int_V \left(\frac{1}{v_j^3} \right) \frac{d\Omega}{4\pi} \right]^{-1/3}, \; (j=1,2,3), \quad (6)$$

where v_j represent three sound velocities that are the eigenvalues of the Christoffel equation involving the single-crystal elastic constants and $d\Omega$ is the element of a solid angle, i.e., $d\Omega = \sin\theta d\theta d\phi$. Since the integration of Eq. (6) is impractical to perform analytically, the integration is evaluated numerically as a procedure outlined by Alers.[12] Using the single-crystal elastic constants for the individual materials considered, values of the mean velocity of sound have been calculated by

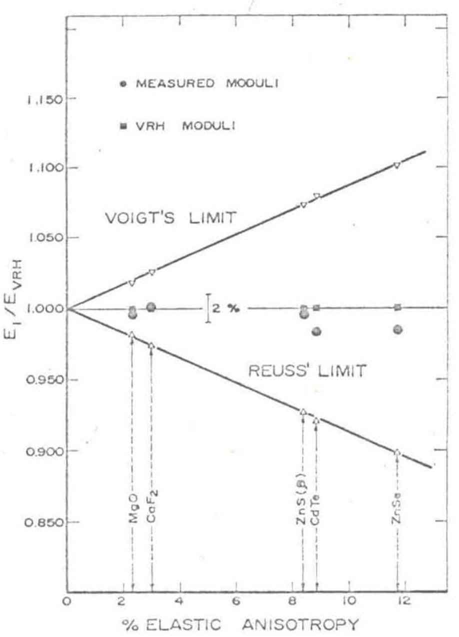

FIG. 1. Comparison between calculated and measured Young's moduli as a function of elastic anisotropy.

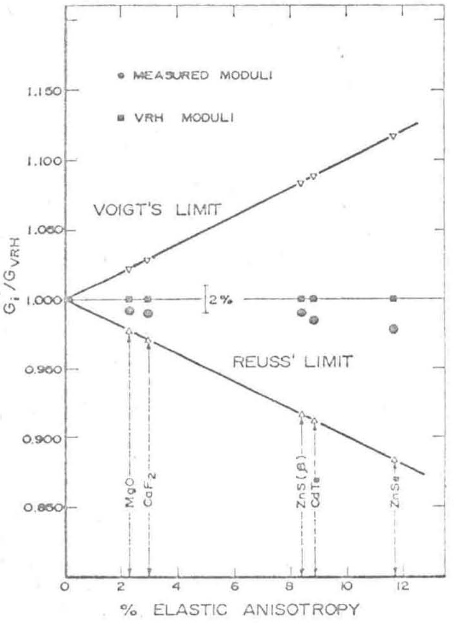

FIG. 2. Comparison between calculated and measured shear moduli as a function of elastic anisotropy.

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[12] G. A. Alers, *Physical Acoustics*, W. P. Mason, Ed. (Academic Press Inc., New York, 1965), Vol. III-B, Chap. 1.

use of IBM-7074 computer. The computed values of v_m are tabulated in Table V and these are compared with values calculated from (i) the VRH moduli and (ii) the measured polycrystalline elastic moduli presented earlier in Table I. Since the polycrystalline elastic moduli are isotropic, such a calculation of the mean velocity of sound from the polycrystalline data is simply done by the use of a Debye expression[13]

$$v_m^* = [\tfrac{1}{3}(1/v_l^{*3} + 2/v_t^{*3})]^{-1/3}, \qquad (7)$$

where v_l^* and v_t^* are the isotropic longitudinal and transverse velocities of sound, respectively, and they are defined by the isotropic longitudinal and shear moduli in the usual way.

It is evident from Table V that the values of the mean velocity of sound calculated from the single-crystal data agree well with that obtained from the polycrystalline data. In other words, the values of v_m^* calculated from Eq. (7) using both the VRH moduli and the actual polycrystalline moduli find in the general agreement with the v_m calculated from Eq. (6). The kind of the agreement observed here supports the earlier conclusion that the VRH approximation gives realistic values of the polycrystalline elastic moduli in terms of the corresponding single-crystal properties.

ACKNOWLEDGMENTS

The writers acknowledge Dr. Edward Carnall, Jr. for providing the Irtran samples. This work was supported by U.S. Office of Naval Research.

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[13] See, for example, T. H. K. Barron, Phil. Mag. 7, [46] 720 (1955) and also Ann. Phys. 1, 77 (1957).

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Behavior of Saturable-Absorber Giant-Pulse Lasers in the Limit of Large Absorber Cross Section

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Using a rate-equation model, it is shown that the behavior of the saturable-absorber giant-pulse (SAGP) laser can be adequately described in terms of two parameters for values of the ratio of absorber to laser absorption cross section $\sigma > 200$: n_{ai}' the normalized initial inversion and $\sigma\tau_s$, where τ_s is the normalized absorber relaxation time. In the general case, specification of n_{ai}', σ, and τ_s is required. Theoretical curves of the giant-pulse output power, energy, and rise- and falltimes are presented. The results are applicable in particular to SAGP lasers employing organic-dye absorbers.

INTRODUCTION

Recently, the saturable-absorber giant-pulse (SAGP) laser has been the subject of intensive study.[1–16] SAGP lasers might be classed into two types, depending on the nature and distribution of absorber centers in the laser cavity. The more common device uses an organic-dye absorber, which is physically separated from the amplifying medium. Such dyes have absorption cross sections which are typically 10^3 to 10^5 times larger than that of the laser centers. In the other type of SAGP laser, the absorber is uniformly distributed throughout the amplifying medium, e.g., Nd^{3+} glass co-doped with UO_2^{2+},[2] color centers in Nd^{3+} glass[3] and Ho^{3+} glass co-doped with Fe^{2+}.[4] There have also been indications of self Q-switching in ruby co-doped with Ti and Fe.[5] The absorber parameters in the latter systems are, as yet, largely undetermined.

In earlier publications[6,7] a theory of SAGP lasers was formulated in terms of three parameters, n'_{ai} the normalized inversion prior to Q switching, σ the ratio of absorber to laser cross section, and τ_s the absorber lifetime normalized to the cavity photon lifetime. It is the purpose of this paper to show that for sufficiently large σ, the SAGP laser behavior can be adequately described by only two parameters, n'_{ai} and the product $\sigma\tau_s$. The range of validity of this description is examined in detail and is shown to be a good approximation for

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